Multiphysical Computations in Fluid Dynamics and Electromagnetics

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Abstract

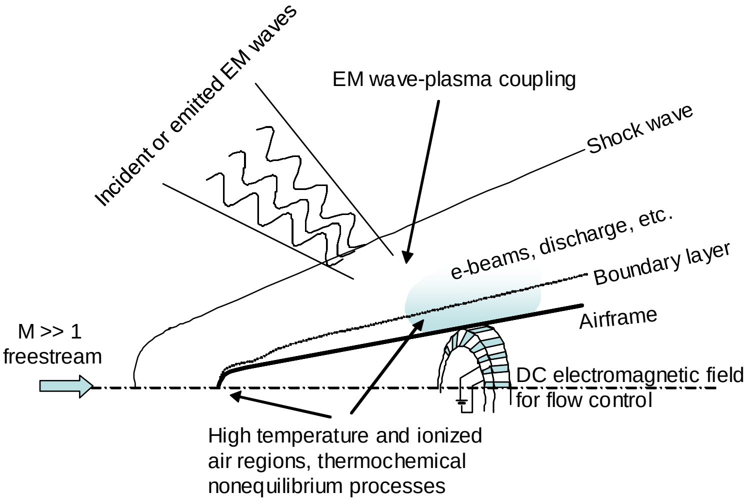
There has been a phenomenal growth in the use of multi-disciplinary computations in technology intensive areas of engineering in recent times. Interaction between fluid flow and electromagnetic fields appears in nuclear engineering, semiconductor processing, metallurgy and is a dominant mechanism in astrophysical gas dynamics. Applications in aeronautics have regained prominence since the early 90s, in studies of magneto-plasmadynamic propulsion, hypersonic shock wave control, plasma enhanced combustion, boundary layer control, and electromagnetic wave propagation in ionized gases. While these concepts have been conceived several decades ago, computational tools to study them with a comprehensive set of physical models for realistic conditions and complex geometry are only becoming available in the present time. There is growing confidence in the accuracy of these computations, and promise that a handful of high-impact applications of this technology will remain relevant despite inherent limitations (low conductivity of air, high recombination rate of ionized species, etc.,) for the foreseeable future. This paper summarizes ongoing research in the computation of such problems with emphasis on higher order accurate, high performance computing tools developed at HyPerComp Inc.

1. Introduction

The sciences of fluid flow and electromagnetics (EM) evolved essentially independent of each other, despite the many similarities that are present in their mathematical description. The interaction between a conducting fluid and an applied magnetic field was identified by Michael Faraday in the early 1830s. This interaction was very feeble (considering that the fluid was water in the river Thames, and the magnetic field was that of the earth,) and the subject matter rested thus for close to a century. Rapid progress was made since the 1930s in understanding this interaction following Hannes Alfven's remarkable discovery of hydromagnetic waves that are essentially responsible for sunspots and other astrophysical phenomena. The name Magnetohydrodynamics (MHD) was coined by Alfven and denotes the flow of conducting fluids in magnetic fields. An independent discipline of Electrohydrodynamics (EHD) was developed many years later, with emphasis on the electrostatic phenomena bringing about changes to fluid bulk and surface properties, much of the classical theory owing its origin to G.I.Taylor. While MHD and EHD phenomena pertain to the limit of slowly varying electric and magnetic fields, a much finer grain interaction is possible when ionized gases are subject to high frequency electromagnetic fields.

Tremendous physical complexity is inherent in coupled fluid and electromagnetic phenomena. Multiphase flow, phase change, particle tracking, effects of compressibility, thermal and chemical nonequilibrium, turbulence, presence of solid walls, and ferromagnetic phenomena sometimes all occur together in a coupled fluid-EM problem. In addition, there are issues pertaining to time and length scales associated with these processes. For instance, electromagnetic waves travel at the speed of light, while waves in fluids are closely allied to the local speed of sound. Chemical reactions, thermal nonequilibrium, phase change, and particle motion all have their own distinct time scales, which usually differ from one another by several orders of magnitude. MHD and plasma flows also exhibit a wide variety of length scales. Liquid metal flow in strong magnetic fields contains extremely thin Hartmann layers where most of the induced current is carried, while the (even smaller) Debye length determines the extent of charge imbalance in plasmas and ionic fluids.

In this paper, we present a description of the state of practice of numerical techniques applied to contemporary interests in the coupled system of equations describing fluid flow and electromagnetics. Figs 1 and 2 show typical scenarios in which these fields have been studied in aeronautics, nuclear engineering and metallurgy. We begin by describing engineering applications which require such coupled multiphysical analysis. We then describe physical and numerical models and their limitations, with a note on the use of adaptive, multi-order accurate techniques in these situations, and conclude with a few results from recent studies. The focus of our presentation is in the general area of MHD.



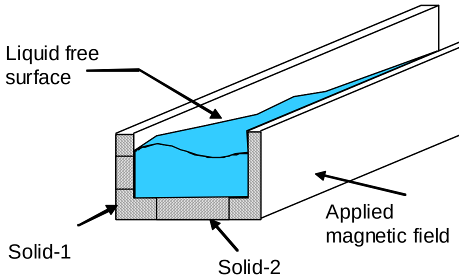
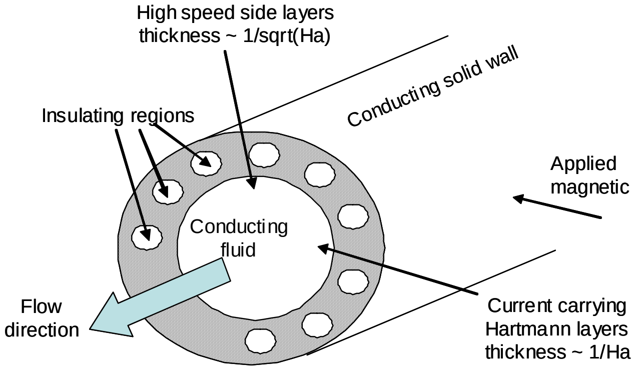


Figure 1: External flow applications (a,left): Hypersonic flow with plasma control, (b,right): Free surface flows of liquid metal in metallurgy and fusion reactor blankets



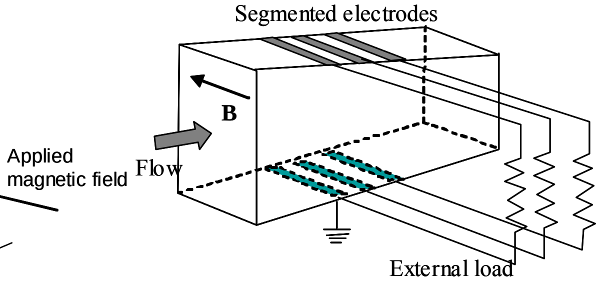


Figure 2: Internal flow applications (a,left): MHD Pipe flow, (b,right): MHD power generator and accelerators

2. Engineering applications

We place the engineering applications which require the multiphysical modeling of electromagnetics and fluid flow in the following four categories, the first three of which involve some form of flow control derived from EM interactions:

Incompressible flow: The flow of liquid metals (e.g., Lithium-Lead) in the presence of strong magnetic fields is a major issue in the design of blanket modules in proposed fusion reactor concepts. Liquid metals are very good conductors of electricity and suffer a large pressure gradient and flow non-uniformities while flowing through magnetic fields. MHD is used in the continuous casting of Aluminum and Steel and in the contact-free processing of these materials. MHD effects have recently been used in forming high quality crystals, particularly those of Silicon, for the semiconductor industry. There have been some popular implementations of flow meters based on MHD principles. Incompressible MHD is the principal mechanism in the flows of molten metal in the earth's core, and in understanding the formation of sunspots and various astrophysical phenomena. The text books by Moreau [1] and Davidson [2] provide further details on these subjects.

Compressible flow and weakly ionized plasma: Ground based MHD power generation using combustion gases was given a great deal of attention in the 1960s. However, interest in this technique declined following the development of more efficient methods. In the 1950s, it was proposed that MHD be used in the reduction of total pressure loss and stagnation point heat transfer in hypersonics, reducing skin friction and providing flow acceleration, all of which are highly desirable in high speed flight. The practical limitation in these concepts comes from the extremely low values of electrical conductivity of air and the feeble nature of MHD effects at conditions of interest. Refer [3] for a summary of the status of "global" flow control using plasma/MHD. There has, however, been a resurgence and some intriguing implementations of these ideas recently. "Localized" gaseous discharges and electromagnetic effects are showing promise in controlling the separation and transition of boundary layers (Ref. [4],[5]). Plasma effects have also been seen to enhance combustion processes, reduce ignition delay, and reduce harmful emissions from engines (Ref. [6],[7],[8]).

Electrohydrodynamics, ionic fluids and microflows: Plasma effects in the absence of magnetic fields, have been suggested for external flow control (Ref. [9]). Electric fields have also been demonstrated in using actuators made of ionic polymer gels and in the control of micro fluidic flows. This is a subject of much current activity, due to potential applications in biomedical devices. Ref. [10] presents some initial observations on such phenomena.

Electromagnetic waves in plasmas: The propagation of radio waves and other forms of electromagnetic radiation is strongly influenced by the presence of plasmas and ionized gases. This results in difficulties with telemetry, greatly modified radar cross sections, communications black-outs in re-entry flows and so forth. Refs. [11,12] describe some attempts at modeling the influence of hypersonic ionized air on EM-waves. While most numerical models consider the influence of the plasma on the EM waves, a full coupling including the effect of the EM-waves on the plasmas is not usually attempted. This would require a wide multi-scale coupling across physics and is often substituted with semi-analytical models. Plasmas are used widely in semi-conductor processing and in nuclear engineering, subject to heating by radio-frequency waves. Two-fluid, drift diffusion models are common practice in these applications. Energy transfer processes into the plasma from the EM radiation is usually considered, while the reverse coupling is omitted for simplicity.

3. Forms of the governing equations and coupling modes

There are a large number of physical processes which come under the subject matter of this paper, as discussed in the earlier section. In this section we first discuss the formulation of the electromagnetics part of the problem from the perspective of MHD and eddy current analysis. Important parameters which characterize the study of MHD flows and plasmas are:

Magnetic Reynolds number: $\mathrm{Re}_m = L\,U\,\mu_m\sigma$ (ratio of induced field to applied field)

Hartmann number: $\mathrm{Ha} = BL\sqrt{\dfrac{\sigma}{\mu_m}}$ (ratio of EM force to viscous force)

Interaction parameter: $\mathrm{N} = \dfrac{B^2 L\sigma}{\rho U}$ (ratio of EM force to inertia)

Debye length: $\lambda_D = \sqrt{\dfrac{\varepsilon_0 kT}{ne^2}}$

In the above expressions, L is the characteristic length in the flow, U is a characteristic velocity, ε_0 and μ_m are the free space permittivity and magnetic permeability, σ is the electrical conductivity, n and e are the electron number density and the electron charge, B is the applied magnetic field, T is the temperature and ρ is the fluid density.

MHD flows are distinguished primarily on the basis of the magnetic Reynolds number Re_m. Fluid equations require the current density J and the local value of the magnetic field B in order to compute the EM (Lorentz) force and Joule heating effects. For the general case (arbitrary Re_m), these quantities may be obtained from Maxwell's equations reduced under the MHD assumption that the displacement current is negligible. We then have:

$$\vec{J} = \frac{\nabla \times \vec{B}}{\mu}\text{, and } B \text{ is obtained from } \frac{\partial \vec{B}}{\partial t} - \vec{\nabla} \cdot \left(\vec{V}\,\vec{B} - \vec{B}\,\vec{V}\right) + \vec{\nabla} \times \left(\frac{1}{\sigma}\left(\vec{\nabla} \times \frac{\vec{B}}{\mu_m}\right)\right) = 0$$

If Re_m is much smaller than 1 as in most engineering applications, the magnetic field induced by virtue of fluid flow will be negligible. B is every where the same as that imposed upon the flow (assuming that the materials are not magnetic or dielectric,) and a much simpler equation is obtained for the current density in terms of a scalar potential φ (A Poisson-type equation for φ results from the requirement that J is divergence free, and must be solved iteratively):

$$\vec{J} = \sigma\left(-\vec{\nabla}\varphi + \vec{V} \times \vec{B}\right) - \frac{\omega\tau}{B}\left(\vec{J} \times \vec{B}\right) \text{ where } B \text{ is prescribed and } \omega\tau/B \text{ is the Hall parameter}$$

We present in figure [3] an illustration of the effect of these formulations on a frequently referenced MHD shock-tube problem devised by Brio and Wu [13]. The classical sequence of shock-contact surface-rarefaction seen in a pure gas dynamic shock tube shows the same wave structure with some smearing of all the interfaces in figure 3a where Lorentz force and Joule heating terms are added as source terms to the conservation equations. Figure 3b shows the solution to the full system of coupled Navier-Stokes – Maxwell equations for a similar situation, where we see the formation of compound waves that are unique to this formulation, and can never be captured using the inductionless approach.

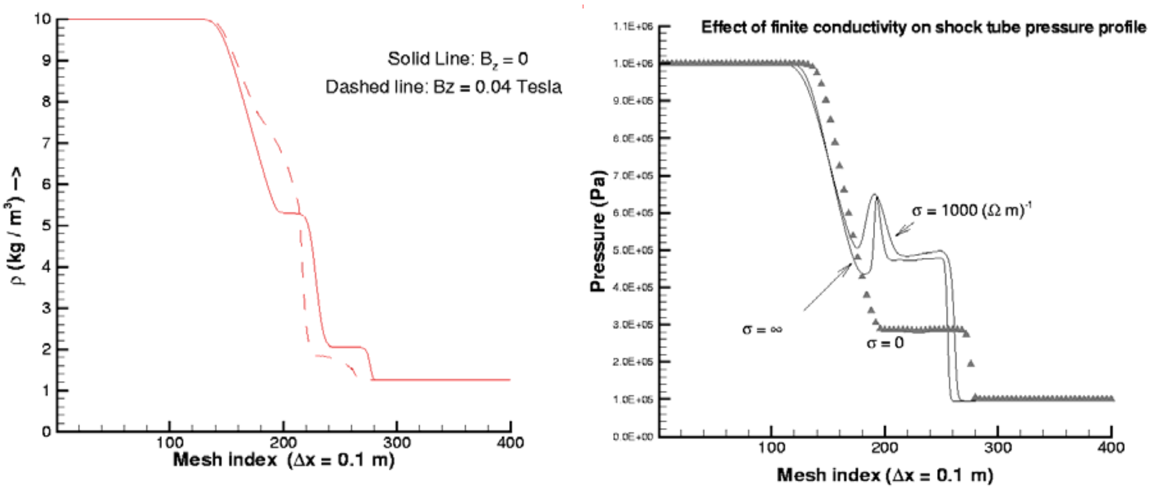


Figure 3: Density profile in a shock tube modeling MHD as source terms (a, left) and using the full wave coupled system with dissipative terms due to finite conductivity (b, right)

It is of interest to note that a much richer variety of analytical formulations are in popular vogue in the study of eddy currents (used in non-destructive testing and evaluation of plasma-structure coupling,) and in magnetostatics. These have been summarized in fig [4]. Note that these formulations distinguish between conducting and insulating regions, and use a number of scalar and vector potential functions defined in the figure.

Formulation	variable location	in the current carrying region	current free region
$\mathbf{A} - \phi$	$\mathbf{A}\phi$; $\mathbf{A}$	$\text{curl}\,(\nu\,\text{curl}\,(\mathbf{A})) = -\sigma\,(\partial \mathbf{A}/\partial t + \text{grad}\,\phi)$	$\text{curl}\,(\nu\,\text{curl}\,(\mathbf{A})) = 0$
$\mathbf{A} - \phi - \Omega$	$\mathbf{A}\phi$; Ω	$\text{div}\,(-\sigma\,(\partial \mathbf{A}/\partial t + \text{grad}\,\phi)) = 0$	$\text{div}(-\mu\,\text{grad}\,\Omega) = 0$
$\mathbf{A}^* - \Omega$	$\mathbf{A}^*$; Ω	$\text{curl}\,(\nu\,\text{curl}\,(\mathbf{A}^*)) = -\sigma\,(\partial \mathbf{A}/\partial t)$	
$\mathbf{T} - \Omega$	$\mathbf{T}\Omega$; Ω	$\text{curl}(\text{curl}\,(\mathbf{T})/\sigma) = -\partial/\partial t\,(\mu(\mathbf{T} - \text{grad}\,\Omega))$ $\text{div}\,(\mu(\mathbf{T} - \text{grad}\,\Omega)) = 0$	
$\mathbf{E} - \Omega$	$\mathbf{E}$; Ω	$\text{curl}\,(\nu\,\text{curl}\,(\mathbf{E})) = -\sigma\,(\partial \mathbf{E}/\partial t)$	

Legend:

$\mathbf{A}$: Magnetic vector potential $\mathbf{B} = \text{curl}\,(\mathbf{A})$

ϕ : Electric scalar potential $\mathbf{E} = -(\partial \mathbf{A}/\partial t + \text{grad}\,\phi)$

Ω : Magnetic scalar potential $\mathbf{H} = -\,\text{grad}\,\Omega$

$\mathbf{A}^*$: Modified $\mathbf{A}$: $\mathbf{A}^* = \mathbf{A} + \int \text{grad}\,\phi\;dt$

$\mathbf{T}$: Current vector potential $\mathbf{J} = \text{curl}\,(\mathbf{T})$

μ : mag. perm., $\nu = 1/\mu$, σ = elec. cond.

Separate regions for storing variables : $\mathbf{J} \neq 0$; $\mathbf{J} = 0$

Figure 4: A summary of various formulations used commonly in the modeling of eddy currents

Equations describing the conservation of mass, momentum and energy for a mixture of gases in thermochemical nonequilibrium can be formulated using classical techniques (Ref. [14]-[16] for a detailed presentation). Electrical conductivity and other transport properties are deduced as local functions of species mass fractions and flow conditions. In multiphase flow, these properties are given as functions of temperature in each material. Level set or the Volume of Fluid (VOF) techniques may be used to capture the free surface and account for phase change at interfaces. Care must be taken to ensure that fluxes such as current and heat are continuous across material interfaces and the appropriate jump conditions are implemented where contact resistance exists. Refs [17],[18] summarize some of our prior work in this direction in more detail. In addition to these effects, electrostatics affects surface properties such as surface tension, adhesion and contact angles. To a large extent this interaction is only treated in an empirical manner.

In flows with high Hartmann number and large interaction parameters, very thin layers known as Hartmann layers develop in the fluid region where the local magnetic field has a non-zero component perpendicular to the wall. In these layers a very large amount of current passes through a spatial dimension of a few microns. Problem specific semi-analytic treatment of these layers is possible, while a truly general solution will have to rely on comprehensive modeling, such as in Ref.[18].

Plasmas and ionic liquids and gels are characterized by spatial dimensions of the order of the Debye length defined earlier. This dimension is much smaller than the characteristic length in the flow. As in the case of the Hartmann layers, approaches to modeling these layers resort to imposing an analytical treatment of this layer, thereby not resolving it in the numerical method, or by actually resolving this layer with computational cells. The importance of this layer is that these layers can support a net non-zero quantity of electric charge, and produce fields that are responsible for bulk plasma properties such as current, Joule heating and so forth. Refs. [10],[19],[20] contain further details on such flows.

4. Challenges in physical and numerical model development

Discharge modeling and the drift diffusion approach: In modeling discharge physics, a vast disparity in the time and length scales emerges and necessitates detailed kinetic modeling of the plasma. Various authors have explored the drift diffusion approach in modeling aerodynamic plasmas (see e.g., Surzhikov and Shang [20]). In gaseous discharges, Joule heating of the bulk plasma takes place within microseconds of plasma ignition, as the electric field and current density patterns are established. The source of heat is a non-neutral region created a few Debye lengths away from electrode surface. The drift diffusion equations are relevant and useful in such situations. Computational time step permitted in solving the drift diffusion equations being extremely small, efficient implicit techniques are needed if these phenomena are to be studied in complex geometries at realistic pressures and if the effect of a magnetic field is to be included. A very similar problem arises in the modeling of ionic polymer gels, where there is often no alternative to resolving the Debye length.

Non-strictly hyperbolic nature of PDEs: The coupled Navier-Stokes and Maxwell equation set discussed earlier, is mathematically termed as a “non-strictly hyperbolic system”. This implies that there is inherent ellipticity in these equations, emerging from the requirement that the computed magnetic field must be divergence free. Purely time-marching algorithms must be modified to account for this effect. Powell (see e.g., [21]), upon recognizing one of the eigen values as identically zero in the coupled equations, introduced a source term, thus making the equation set non-conservative, but solvable using conventional Riemann solver strategies. Other approaches (Balsara [22], MacCormack [23]) have preserved the conservative nature of these equations, and added additional equations in order to enable a flux splitting solution. The computed magnetic field must be post-processed using a projection method in order to make it divergence-free.

Uncertainty in boundary conditions: As in all problems dealing with electromagnetic phenomena, the location of an external (far-field) boundary plays a key role in the numerical solution. Higher order absorbing boundary conditions have been developed for modern finite volume schemes, that are equally applicable to the coupled equation set in MHD as well as Maxwell equations in their pure form (see e.g., Hall et al [24]).

Interface jump conditions: Very often, conservation of current, species and energy must be enforced at material boundaries in a computational domain. In plasmas and MHD, Debye layers are formed at electrode surfaces which are too slender to resolve in practical numerical computations. Various researchers have resorted to modeling these layers with appropriate jump conditions. A first principles approach using conservation laws for charged species at the interface tends to be computationally expensive. Contact resistance is an important aspect of MHD computations, and is more complex to address than the thermal contact resistance problem in conventional heat transfer. The difference comes from attempting to enforce a jump condition involving a continuous normal component of current and a discontinuous electric potential at the interface. The electric potential is solved as a solution to an elliptic equation with no time marching facility. This leads to difficulties in maintaining numerical stability and is a challenge in unstructured meshes.

Resolving length and time scales (role of higher order methods): Adaptive higher order numerical methods provide an effective way to resolve issues in capturing physical phenomena with disparate length scales. The formal order of the numerical scheme can be varied in the domain to suit the need of the modeling problem. This has been carried out effectively in pure electromagnetics and fluid mechanics by various researchers and in the code implementations at HyPerComp.

The role of Hartmann layers and conducting walls: Hartmann layers are thin regions of the flow where the normal component of the applied magnetic field to the wall is significant. These layers are unique to MHD flows. They are fundamentally different from conventional boundary layers because small perturbations in Hartmann layers affect the core flow to the leading order. As a rule of thumb, numerical errors which may cause these perturbations are amplified by the square of the Hartmann number, which can be disastrous in fusion relevant liquid metal flow problems, where the Hartmann number is of the order 10^4. This becomes an extremely serious issue on non-orthogonal meshes and in complex geometries. We have developed a pioneering technique to effectively solve this problem and intend to publish it in the coming year [25].

Long period wave propagation: All numerical schemes designed for compressible plasma/MHD have their basis in being able to capture wave propagation over long distances and time periods. The following section will describe higher order accurate numerical schemes that we have used and comment on other possibilities.

Conservation issues and code integration: Accuracy in free surface capture, enforcing conservation of electric current, maintaining energy conservation in the multiphysical coupling process and enabling a seamless coupling across physical disciplines are some additional challenges that are encountered. We have preferred the level set technique in free surface capture, due to its easy extension to complex meshes, accuracy of surface tension prediction and the ability to handle very complex free surface shapes. Mass conservation haunts the usage of the level set technique, but is much less of a concern when using the higher order schemes described in the next section. Conservation of electric current, mass and momentum can be enforced explicitly using penalty functions, an easy extension to Galerkin type schemes. Multiphysical coupling using a CAD-centric approach is becoming very popular in the present day. Most commercial software enable this in some form. However, these tend to be generic approaches that will address simple problems with weak interactions and benign flow conditions. There is much scope for the development of a problem specific code coupling procedure. HyPerComp is currently investigating this for the case of coupled thermal-structural-MHD-neutronics coupling as encountered in fusion blanket modules.

5. Higher order accurate numerical schemes

Higher order accurate numerical methods bear the potential to reduce computational burden by decreasing the number of unknowns for a given problem while retaining the same level of accuracy. Finite difference schemes, spectral methods and Galerkin-type methods are the most popularly used procedures to obtain higher order accuracy. Each of these methods attempts to construct a numerical solution to an arbitrary problem by projecting the solution into a subspace of functions which guarantee that the local error produced in each computational cell (for a convergent numerical procedure) is of the order O(h^p) where h is a measure of the cell size and p is the so called "order" of the scheme. When the underlying accuracy of a numerical scheme increases, fewer mesh cells are needed for the same error level.

We express a bias in our work favoring the discontinuous Galerkin (DG) method which, as opposed to the traditional Galerkin method (such as the FEM,) uses basis functions that are local to a given cell and uses an appropriate flux function at cell faces where solutions can be discontinuous. This method can be implemented as a convenient extension to existing finite volume software, with provisions to accommodate an arbitrary set of conservation laws. The text by Karniadakis and Sherwin [26] may be consulted for a survey of the DG technique and its applications in CFD. HyPerComp's contributions in this area have been published in Refs. [24],[27] and [28]. In a nutshell, the DG scheme expresses an integral conservation law in its weak form upon multiplication by a series of test functions υ^i.

$$\int_{V_{ic}} \frac{\partial \vec{Q}}{\partial t} dV = -\int_{V_{ic}} \nabla \cdot \vec{F}\left(\vec{Q}\right) dV \quad \rightarrow \quad \int_{V_{ic}} \nu^i \frac{\partial \vec{Q}}{\partial t} dV = -\int_{S_{ic}} \nu^i \hat{F}\left(\vec{Q}\right) \cdot \hat{n} dS + \int_{V_{ic}} \nabla \nu^i \cdot \vec{F}\left(\vec{Q}\right) dV$$

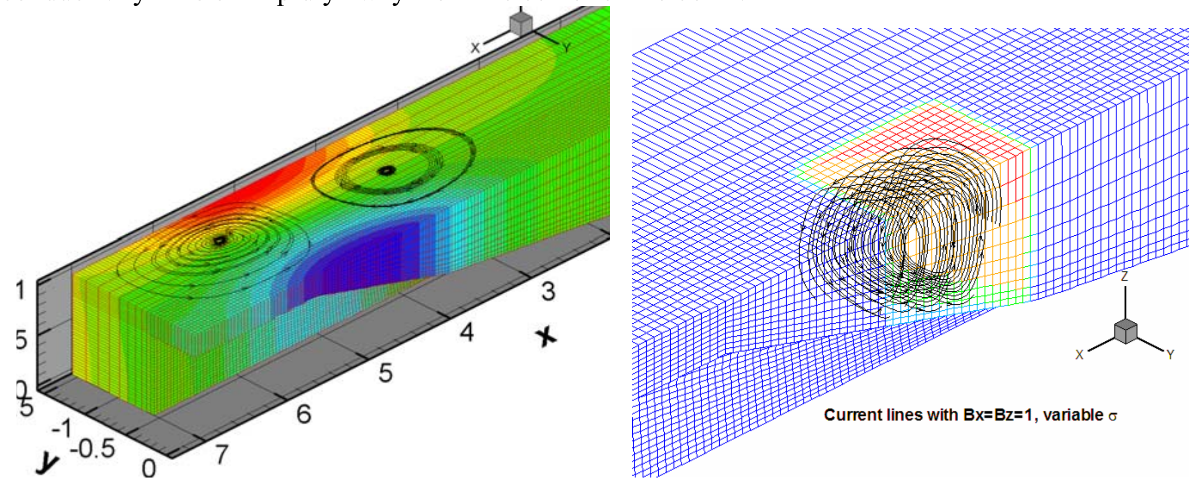
In the Galerkin approach, these same functions are used as a set of basis functions in expanding the vector of physical quantities Q within each cell in terms of a set of unknown coefficients. A series of algebraic equations is obtained and solved to advance the solution to the next time level. Accuracy and efficiency of the procedure is limited in the case of non-linear equations, wherein the flux F is a nonlinear function of the conserved quantities Q. The presence of second derivatives in the conservation laws (dissipative and viscous fluxes) requires a special treatment in order to overcome the limitation that the set of basis functions differentiated twice provides an inconsistent representation of the physical quantity in an otherwise hyperbolic conservation law. Often times, a higher order boundary representation is required to obtain a consistently higher order solution. The method has been applied at HyPerComp Inc. to problems in computational time-domain electromagnetics (Ref. Kabakian et al [27],) and to MHD (Ota et al [28]).

6. Sample results

We present here some results from plasma and MHD modeling studies performed at HyPerComp pertaining to high fidelity modeling of MHD on arbitrary geometries in both compressible and incompressible regimes and a sample result from higher order accurate modeling in MHD. Further details on our work on plasma modeling, computational electromagnetics can be seen in [27],[29].

Hypersonic flows with MHD effects

We modeled the hypersonic flow in an inlet designed for and operated at Mach 6, and studied thermochemical nonequilibrium effects when the flow weakly ionized and a magnetic field is applied. In figure 5a we see current lines in a flow where the electrical conductivity is assumed to be uniformly constant every where (10 mho/m) and a local magnetic field is applied. Figure 5b shows a situation where electron beams are applied in a small region of the flow, providing free electrons and a locally enhanced electrical conductivity. The exchange of energy between electrons and heavy particles (N2, O2, NO, etc.) creates a thermal non-equilibrium, as can be seen in the difference between the translational and vibrational temperatures in fig 5c. The e-beam strength was tuned to achieve a peak conductivity value of 10 mho/m. As can be seen, this effect is highly local and the conductivity falls off rapidly away from the center of the beam.



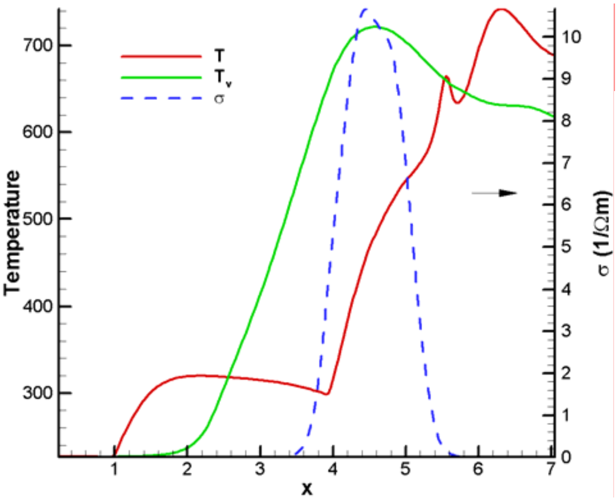


Figure 5: (a, top left) Current lines and electric potential contours for a Mach 6 inlet with a locally applied B-field, (b, top right) Current lines and electrical conductivity contours in a flow with e-beam radiation (c, left) Translational, vibrational temperatures and the electrical conductivity for weakly ionized air with e-beams, as in (b)

Incompressible flow of liquid metals in magnetic fields

In proposed fusion reactor concepts such as ITER, there is need to breed Tritium (one of the fuels of the reactor,) regeneratively from the interaction of neutrons produced in the fusion process and

Lithium or one of its compounds. Among the various concepts to achieve this, has been the dual coolant idea where an alloy of Lithium ($PbLi^{17}$) extracts heat as well as Neutrons from the fusion process while flowing in tubes in a "blanket module". These liquids are excellent conductors of electricity (~10^6 mho/m), and pose many challenges in their numerical modeling. HyPerComp Inc. has developed a pioneering code capability for modeling these flows in complex geometries at very high Hartmann numbers. Figure 6 shows examples of recent results. MHD flows in conducting ducts tend to produce sharp "jets" close to the walls parallel to the applied magnetic field. Very large pressure gradients are created, which are difficult to predict accurately. Further, these flows can possess free surfaces which deform violently during the passage of current. Flow separation, ferromagnetic effects, contact resistance, temperature dependent properties and natural convection are among the many physical phenomena that are modeled in these problems. Ref [17],[18] may be consulted for further details on these studies.

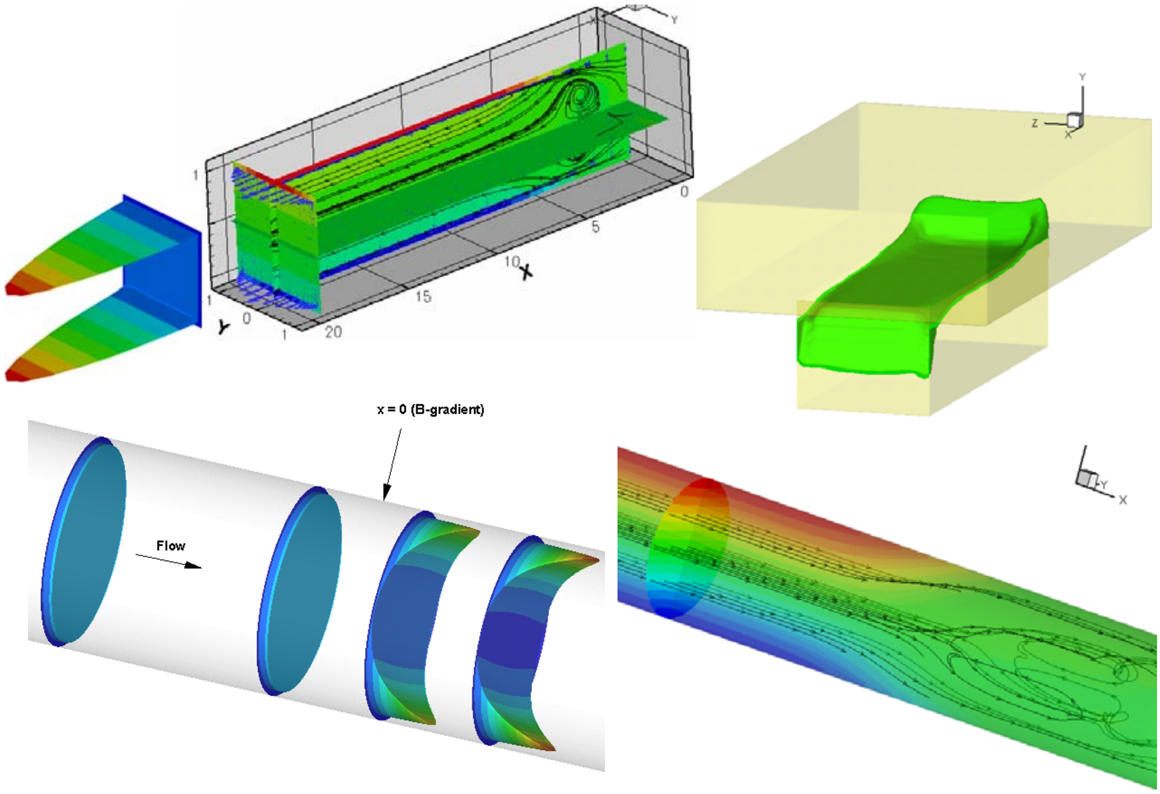


Figure 6: Sample results from liquid metal MHD, clockwise from top left (a) Flow in a square duct with conducting walls at Ha = 10,000 (b) Liquid sloshing caused by MHD forces and electric current injected from an external plasma (c) Streamlines and electric potential contours in a cylindrical duct flow with magnetic field gradient at Ha = 6600, (d) Axial velocity profiles at 4 cross sections of (c)

<u>Higher order accurate MHD vortex modeling</u>

We present results from a higher order accurate solution of the MHD equations using the DG method for the case of an advecting vortex with MHD effects. Balsara [22] devised the following set of initial conditions which result in the vortex translating from the center of a square domain (-5,5)x(-5,5) in a diagonal direction.

$$\rho = 1, P = 1 - \frac{\eta^2\left(x^2 + y^2\right) - \varepsilon^2}{8\pi^2} e^{1-x^2-y^2}, \gamma = \frac{5}{3}, \varepsilon = \eta = 1,$$

$$u = 1 - \frac{y\varepsilon e^{\frac{1-x^2-y^2}{2}}}{2\pi}, v = 1 + \frac{x\varepsilon e^{\frac{1-x^2-y^2}{2}}}{2\pi}, w = 0$$

$$B_x = -\frac{y\eta e^{\frac{1-x^2-y^2}{2}}}{2\pi}, B_y = \frac{x\eta e^{\frac{1-x^2-y^2}{2}}}{2\pi}, B_z = 0$$

Figure [8] shows sample results showing contours of pressure, Mach number and magnetic field components in this flow, and a plot of the logarithm of the error compared with the exact solution against the logarithm of the mesh size. Slopes of these lines provide an estimate of the order of accuracy of the numerical scheme. It can be verified that a p-scheme shows a slope in the vicinity of p+1, the formal order of accuracy of that scheme. Higher order methods are being applied to an ever expanding range of multi-physical problems and are gradually entering the arena of commercial CFD.

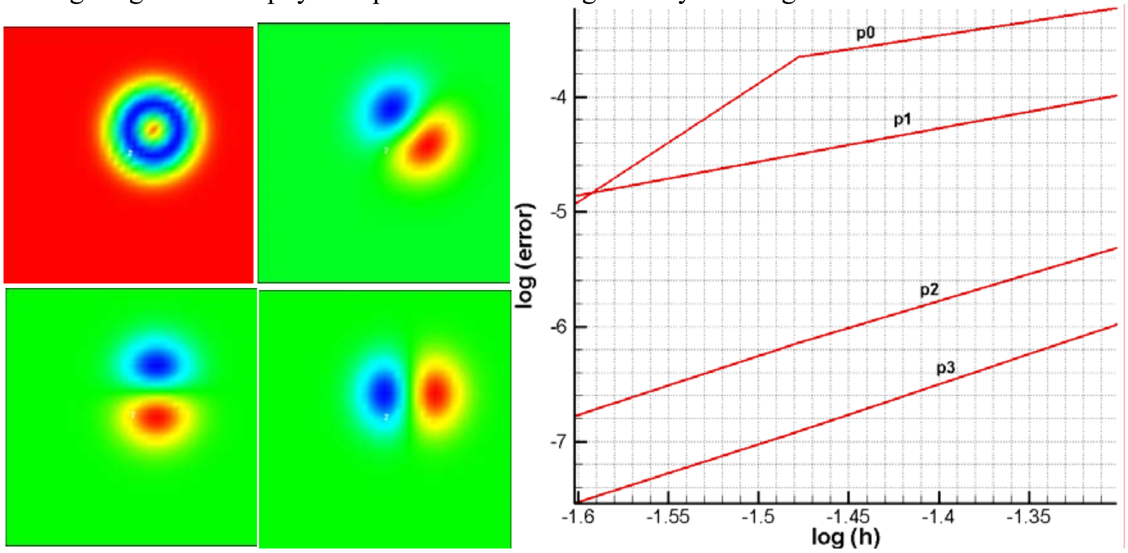


Figure 7: Propagation of an MHD vortex in a uniform mesh, contours of P, M, Bx and By (clockwise from top left,) and error in numerical solution against mesh spacing for various p-orders

7. Conclusion

In this paper, we have attempted to communicate a flavor of the numerical modeling involved in combining the classical disciplines of fluid mechanics and electromagnetics in a single domain. This is a subject rich in physical complexity, and our emphasis has been to highlight numerical and physical hurdles and show sample results in typical situations, so that one is able to appreciate the challenge as well as the scope of these studies. "Localized" flow control using plasma discharges resulting in increased lift to drag ratios for airfoils, and the postponement of boundary layer separation and transition, promises to engage both the modeling community as well as designers in the years to come. There is renewed interest in modeling the propagation of EM waves in ionized gases, and computational efforts are gradually gearing up to meet these challenges. Liquid metal MHD flows will continue to be of interest during the construction of ITER and other major fusion projects worldwide. Microflows, synthetic gel actuators and other applications in MEMS devices rely strongly upon EHD and some MHD effects, and there is every indication that progress in these areas will have great implications in biotechnology in the times to come.

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